



CONTROL DESIGN OF THE TEST MASS RELEASE MODE FOR THE LISA PATHFINDER MISSION



Fabio Montemurro¹, Walter Fichter¹, Markus Schlotterer², Stefano Vitale³

¹ EADS Astrium GmbH, Friedrichshafen, Germany

² ZARM, Centre of Applied Space Technology and Microgravity, Bremen, Germany

³ Department of Physics and INFN, University of Trento, Trento, Italy

The LISA Technology Package (LTP) features two high precision Inertial Sensors (ISs). Each IS encompasses a free floating cubic test mass (TM) surrounded by a capacitive housing. The electrodes, hosted by the housing, form a set of variable capacitors with the TM and are used both for TM sensing and suspension. Moreover, each IS contains a Caging Mechanism (CM) which sustains the TM during launch. Before the science phase of the mission begins, each TM is set free and the control system performs the TM Release Mode.

This poster presents the design of the TM Release Mode: it covers equipment modeling, actuation strategy selection, disturbances estimation, controller law definition, performance results, and stability and robustness analyses.

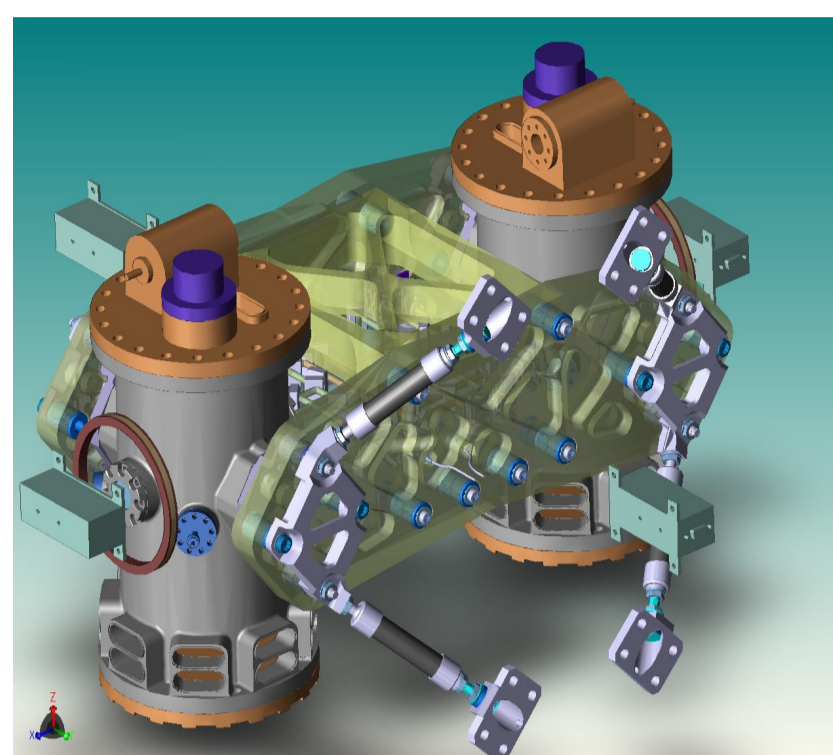
Motivation and Goals

- Successful TM release is crucial for the entire LISA Pathfinder mission
- Stringent performance requirements are set for the TM release
- Manifold and complex interactions between subsystems
- Highly detailed modeling and simulations are required pre-launch to de-risk LISA Pathfinder mission

Equipment Modeling

LISA Pathfinder End-to-end Performance Simulator features, among the others, the following relevant models:

- Inertial Sensor Model, for both electrostatic actuation and sensing
- Functional Model of the Caging Mechanism
- UV Lamp Unit
- 18 DoF non-linear dynamics of the Spacecraft and Test Masses
- On board computer, i.e. Drag-Free and Attitude Control System (DFACS)



Actuation Algorithm for Electrostatic TM Suspension

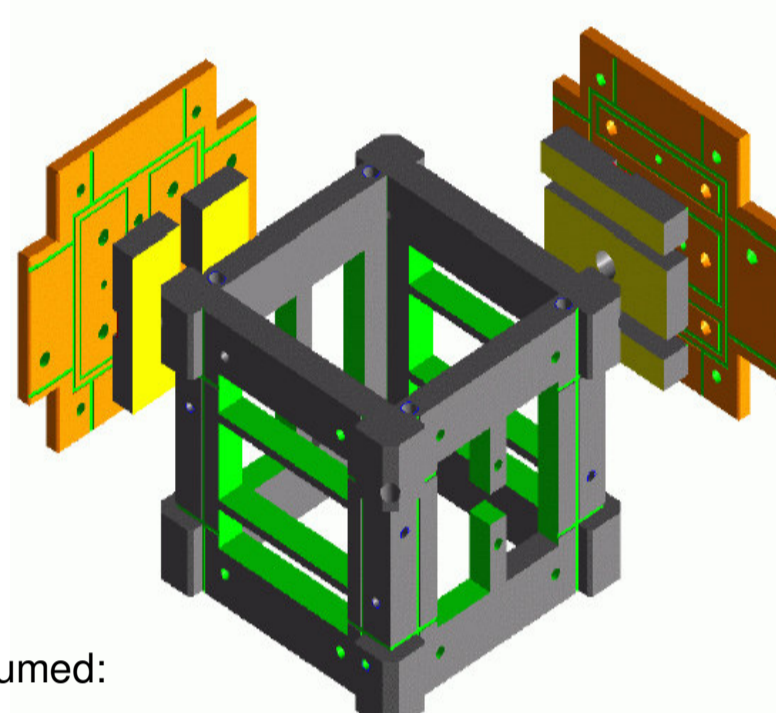
Variable stiffness: An actuation strategy different than for the Science Modes

The actuation forces and torques are obtained by means of voltages applied to the electrodes.

For the TM Release Mode the maximum voltages authority is required.

The expression for a generalized force is given by:

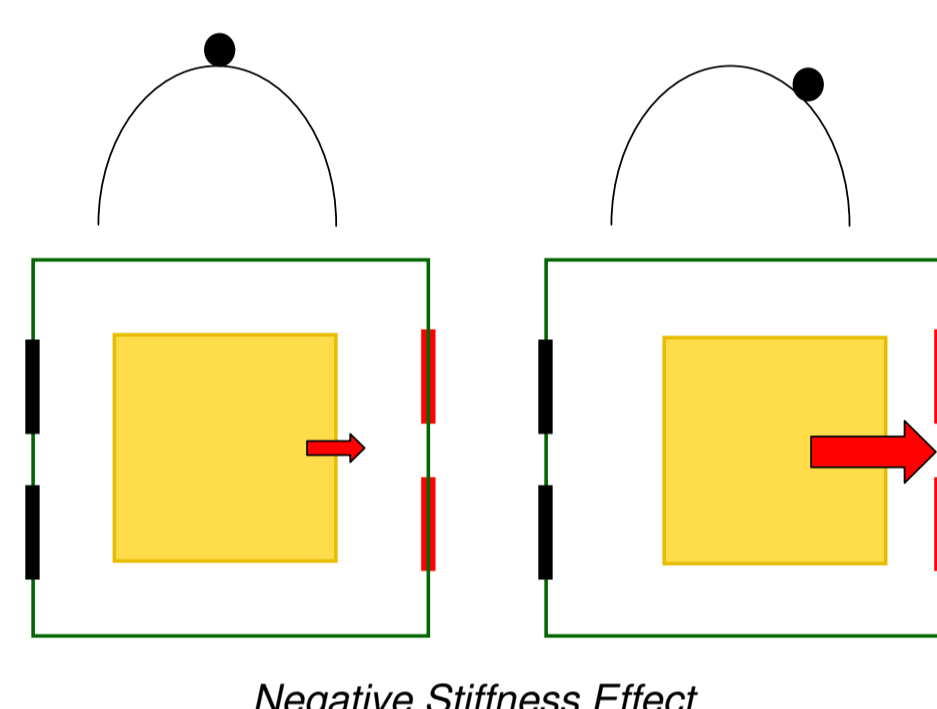
$$\begin{cases} f_q = \frac{1}{2} \sum_{i=1}^{12} \frac{\partial C_i}{\partial q} (V_i - V_{TM})^2 \\ V_{TM} = \frac{1}{c_{tot}} \left(q_{TM} + \sum_{i=1}^{12} C_i V_i \right) \\ c_{tot} = \sum_{i=1}^{17} C_i \end{cases} \quad \begin{matrix} 3 \text{ Equations} \\ 12 \text{ Unknowns (electrode voltages)} \end{matrix}$$



In order to derive a force/torque to electrodes voltages conversion law it is assumed:

- For each axis, only the relevant electrode quadruple is considered
- Actuation voltages are chosen in order to *keep the TM potential to null*
- TM is assumed at nominal centered position as well as with null charge
- Actuation stiffness is negative by definition (*negative spring*):
 - Stiffness increase quadratically with the actuation voltages
 - During the TM release high TM displacement are envisaged

Minimize the stiffness to allow for the largest TM envelope within the IS



$$\begin{cases} f_x = \frac{1}{2} \sum_{i=1}^4 \frac{\partial C_i}{\partial x} V_i^2 = f_{x,desired} \\ f_\theta = \frac{1}{2} \sum_{i=1}^4 \frac{\partial C_i}{\partial \phi} V_i^2 = 0 \\ K_{xx} = -\frac{\partial f_x}{\partial x} = -\frac{1}{2} \sum_{i=1}^4 \frac{\partial^2 C_i}{\partial x^2} V_i^2 \text{ minimized} \\ V_{TM} = \frac{\sum_{i=1}^4 C_i V_i}{\sum_{i=1}^4 C_i} = 0 \end{cases}$$

The Controller Design

Non Linear System

- variable input gain
- control saturation



Sliding Mode Technique

transform a general n^{th} order tracking problem into a *first order* stabilization problem

Definitions

For each DoF the capacitive suspended TM system is defined by: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}$

where it is:

- x_1 position/angle
- x_2 velocity/rate
- x_3 DC disturbance acceleration
- u is the control input, i.e. capacitive actuation
- $B(x)$ is the input matrix. It depends on the state vector since the capacitive actuation is a function of the TM position/rotation (i.e. actuation stiffness)

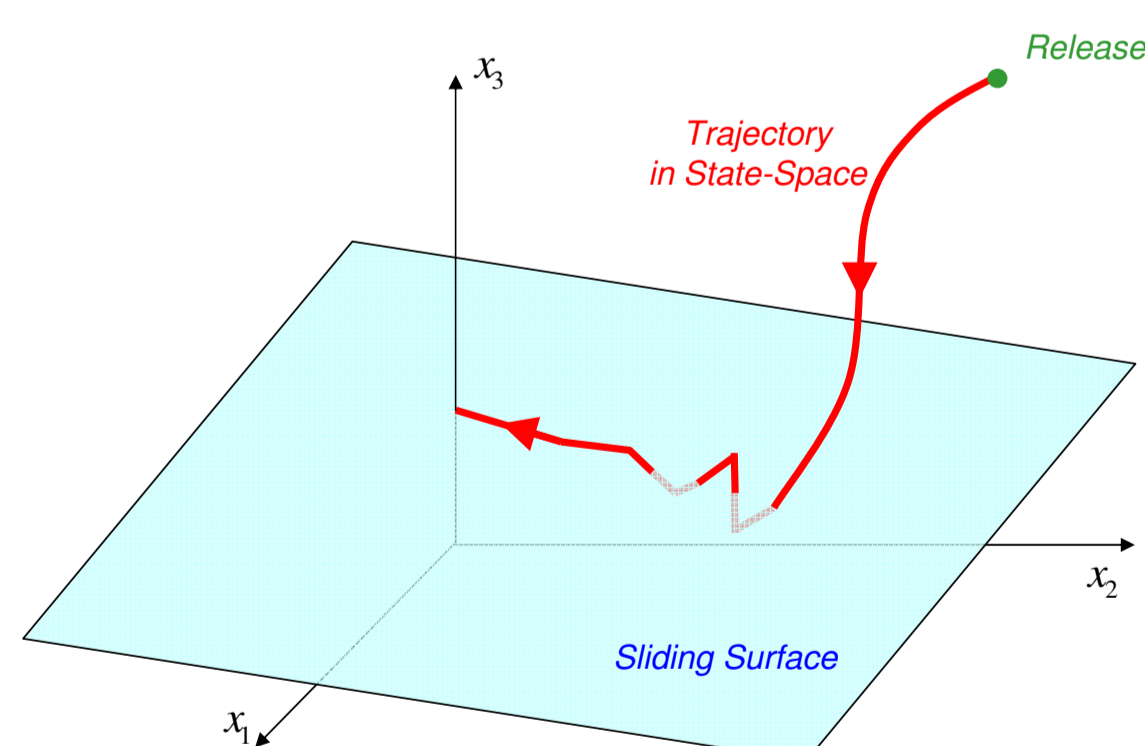
The design model is assumed to be linear like: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

The Sliding Mode method is a *robust* control approach which *explicitly* takes into account:

- system non-linearities
- Not-modeled /unknown disturbances $|\Delta \mathbf{f}(\mathbf{x})| = |\mathbf{f}(\mathbf{x}) - \mathbf{A}\mathbf{x}| \leq \Delta \mathbf{f}_{max}$

The Sliding Condition

The sliding control law drives the plant's state trajectory onto a *sliding surface* and maintains the states on the surface it for subsequent time.



The sliding surface s is a linear combination of the states:

$$s = \mathbf{S}\mathbf{x} = \lambda x_1 + x_2$$

A semi-definite Lyapunov's function of s is defined:

$$V = \frac{1}{2} s^2$$

From Lyapunov's stability theory, the system is stable if:

$$\dot{V} = \frac{d}{dt} \frac{1}{2} s^2 = s\dot{s} \leq -\eta |s| \quad \text{Sliding Condition}$$

where η is a strictly positive constant.

The Control Law

Satisfy the sliding condition while taking into account the differences between the model and the real system

$$u = \hat{u}(t) - k \operatorname{sgn}(s) = -(\lambda x_2 + x_1) - k \operatorname{sgn}(s) \quad k = k(\hat{u}, \eta, \Delta \mathbf{f}_{max}, \mathbf{B}(\mathbf{x}) - \hat{\mathbf{B}})$$

It brings the TM states to the surface: the condition $\dot{s}=0$ is satisfied for the linear design model

Discontinuous (switching) function: It guarantees that the sliding condition is satisfied also with model uncertainties

The sliding surface is selected as *steep* as possible under the **constraint that no control saturation** occurs up to the **maximum theoretical velocity** which is allowable at release

It accounts for:

- Non-modeled disturbances
- Actuation stiffness
- Finite time to reach the surface

In a boundary layer of the sliding surface, the control signal is smoothed out to avoid *chattering*

Time Constant and Comparison with PID controller

Reference Controller: PID featuring the same maximum actuation command as Sliding Controller

Sliding

Phase 1: Reach the surface: $t_{reach} \leq \frac{s(t=0)}{\eta}$

PID

Longest time constant $t_{PID} \sim \frac{10}{\omega_{bandwidth}}$

Phase 2: On the surface:

Exponential convergence

$$x \sim e^{-\lambda t} \rightarrow t_{surface} = 1/\lambda$$

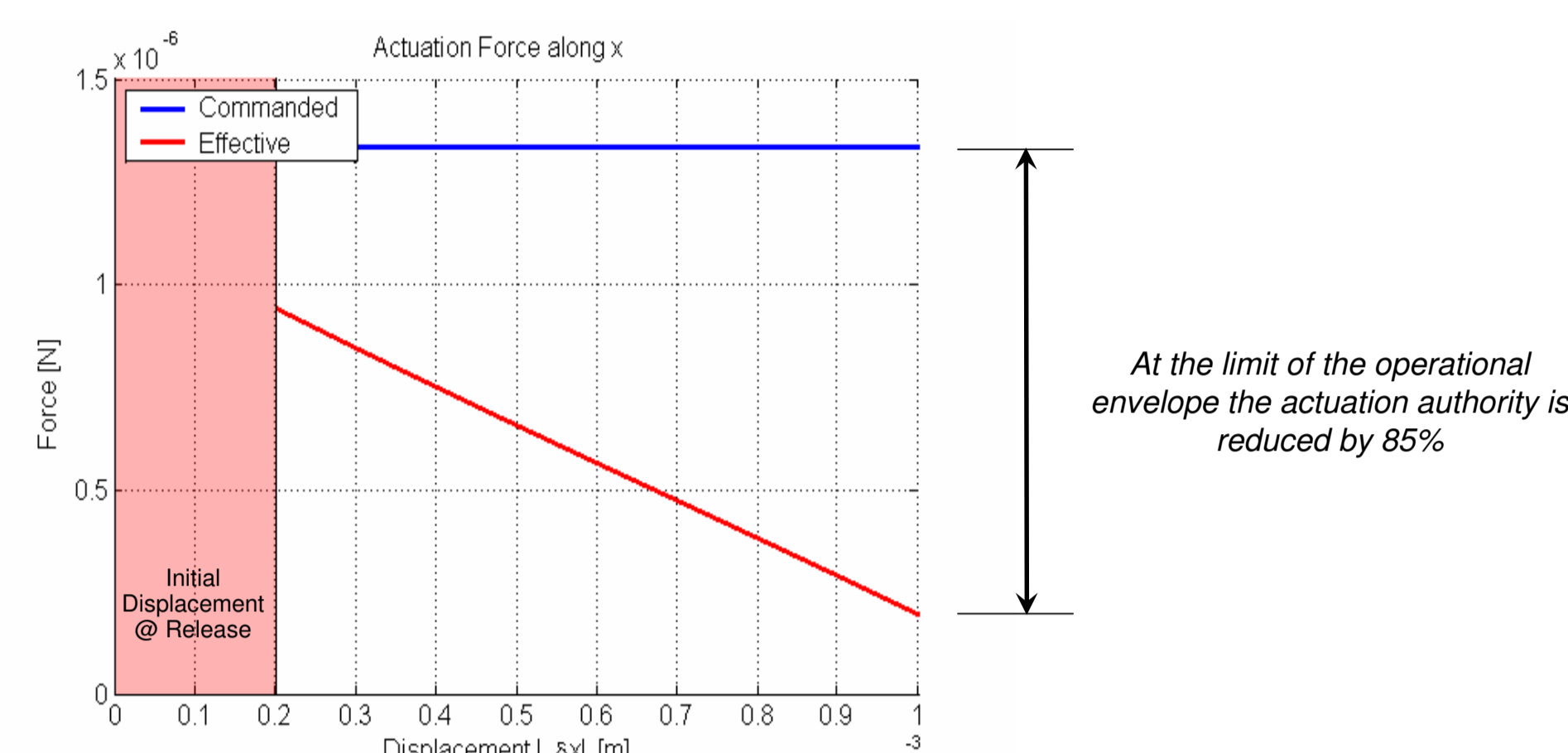
$$t_{SM} = t_{reach} + t_{surface} < t_{PID}$$

A Precious Benchmark for the Design: the Maximum Allowable Initial Velocity

A survey for the maximum TM initial velocity which the IS electrostatic suspension can stand with.

Such velocity can be achieved if the suspension controller applies as soon as possible the maximum force/torque on the TM, in order to draw as much as possible kinetic energy out of the TM itself.

- Only the electrostatic actions are considered to be acting on the TM
- The maximum force/torque is applied as soon as the TM is not caged
- A simplified two degrees of freedom TM model is used; only coupled DoF's are considered (e.g. x and ϕ)
- A *worst case scenario*: initial displacement and velocity have the same sign



By integrating between P_1 and P_2 belonging to the state space (x, ϕ)

$$\begin{cases} m\dot{a}_x = F_{x,max}(x, \phi) \\ I\dot{\omega}_\phi = T_{x,max}(x, \phi) \end{cases} \quad \text{Kinetic Energy Theorem}$$

$$\begin{cases} \frac{1}{2} m v_{x,2}^2 - \frac{1}{2} m v_{x,1}^2 = \int_{P_1}^{P_2} F_{x,max}(x, \phi) dx \\ \frac{1}{2} I \omega_{\phi,2}^2 - \frac{1}{2} I \omega_{\phi,1}^2 = \int_{P_1}^{P_2} T_{x,max}(x, \phi) d\omega_\phi \end{cases}$$

Electrostatic force is a conservative field: the integral does not depend on the path.

The maximum initial velocity is obtained by setting:

- the velocity at P_2 to zero, where P_2 corresponds to the maximum envelope
- P_1 corresponds to the TM position at release

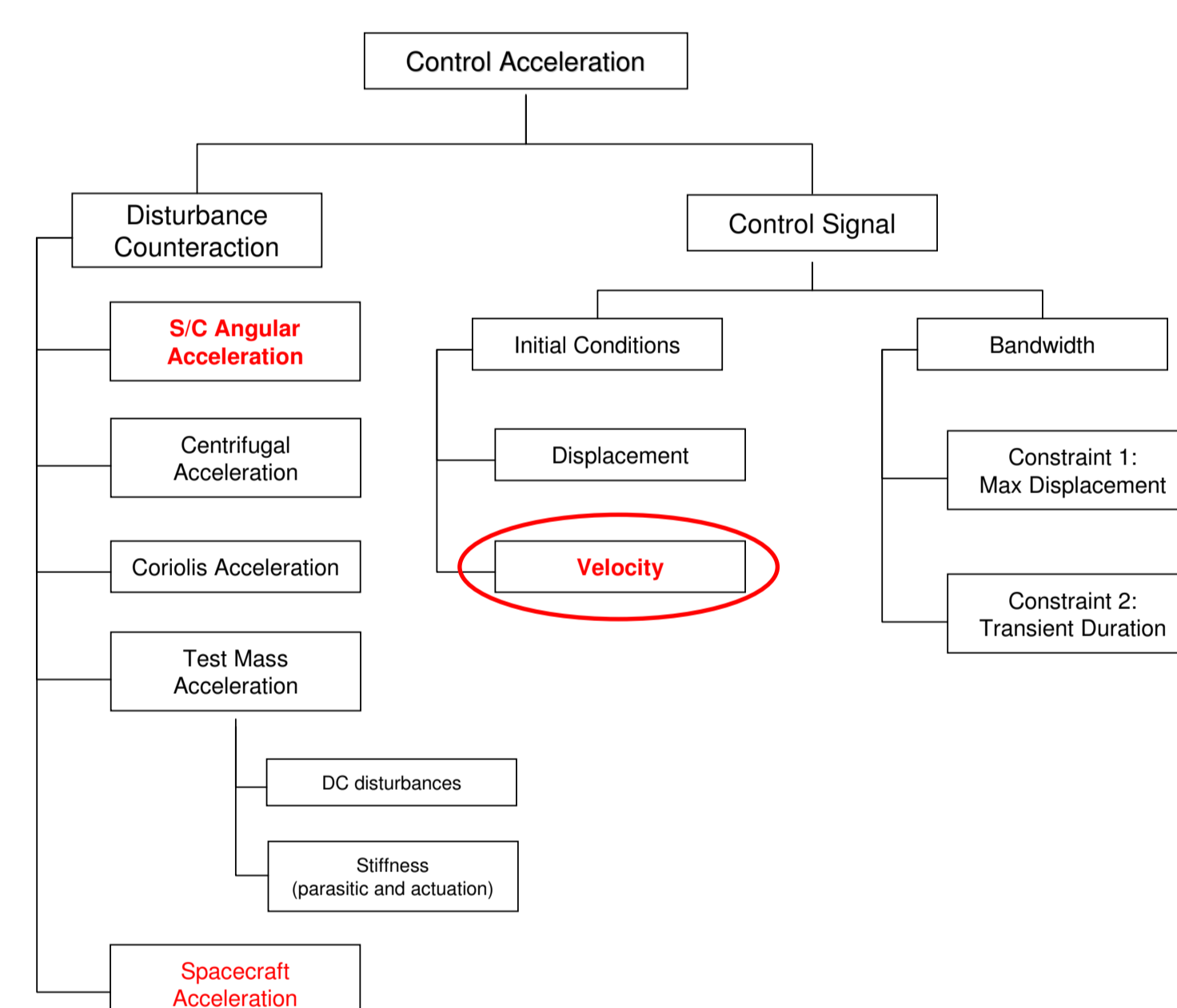
$$\begin{cases} v_{x,1}|_{max} = \sqrt{-\frac{2}{m} \int_{P_1}^{P_2} F_{x,max}(x, \phi) dx} \\ \omega_{\phi,1}|_{max} = \sqrt{-\frac{2}{I} \int_{P_1}^{P_2} T_{x,max}(x, \phi) d\omega_\phi} \end{cases}$$

Control Level Breakdown and TM Disturbances Budget

A detailed control level breakdown prior to the control design.

Since the TM is suspended w.r.t. the S/C, the TM Release Mode features a strong coupling with the S/C dynamics.

$$\mathbf{a}_{rel} = -(\ddot{\mathbf{w}}_{SC} + \ddot{\mathbf{w}}_{SC}^2) \mathbf{r}_{TM} - 2\ddot{\mathbf{w}}_{SC} \dot{\mathbf{r}}_{TM} + \mathbf{u} + \mathbf{d}_{TM} + \mathbf{d}_{SC}$$



- The S/C controller shall strive for an effective reduction of the *disturbances induced* on the TM
- The solar pressure plays a major role in the entire budget
- The requirement on the *TM velocity at release* is a key factor

The Observer Design for the Sliding Mode Controller

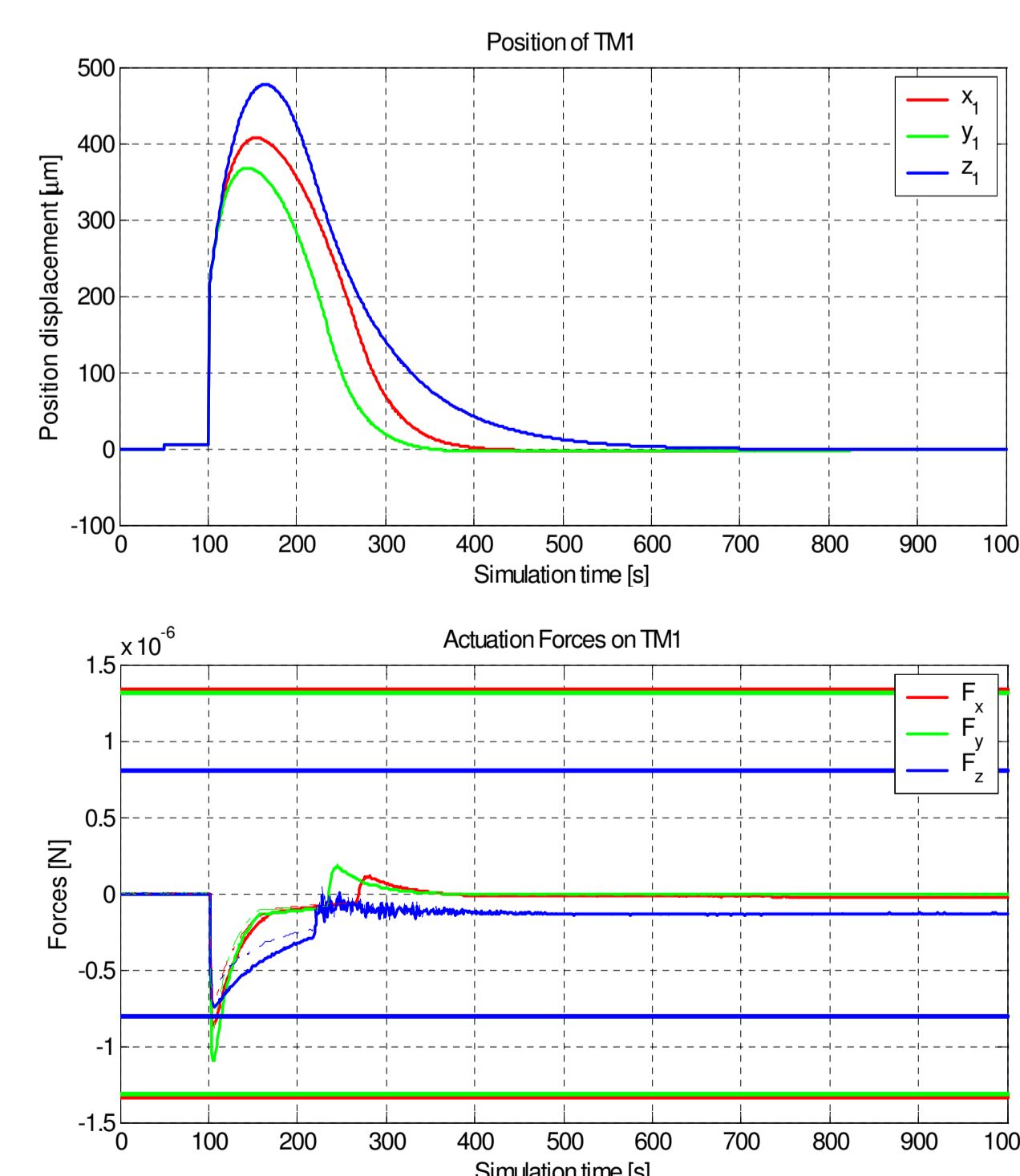
The Sliding Mode control requires an observer for the unmeasured states. A *reduced state observer* is proposed for velocity and disturbances estimation:

- No DC steady error for TM states and Observer states due to non-estimated DC disturbances
- All DC disturbances are estimated (better than an open loop estimation of the solar force only)
- Faster convergence to steady state
- Velocity estimation is fed back to the **Caging Mechanism** for eventual automatic re-caging

Simulation Results

De-caging @ $t = 100$ s
Initial Displacement: 200 μm
Initial velocity: 10 $\mu\text{m/s}$

Maximum Overshoot < 300 μm
Control Accuracy < 2 μm



Advantages w.r.t. PID controller

- Robustness by definition
- Better performance
 - Less overshoot
 - Faster convergence
 - Higher steady state accuracy